ROUND I: Evaluation of algebraic expressions

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If x & y = $x^y + y^x$, find the value of 2 & 3.

2. Evaluate for x = 3, y = -4, z = 2:

$$\left|\frac{xz}{y}\right| - \left(-\left|\frac{-y}{z}\right|\right) - x^2$$

3. If a * b = $a^2 + b^2$, evaluate the exact value of $\frac{(a*b)+2ab}{a+b}$ as a reduced fraction when a = $\frac{1}{27}$ and b = $\frac{1}{9}$.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Bancroft. Doherty. South

ROUND II: Solving linear equations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve for y: $2-(4y+6)+(8y-10)-(12y-14)=16$

2. Solve for x: $\frac{1}{8}(7x+5)-\frac{1}{10}(3x+15)=2$

3. Solve $v=c\left(1-\frac{t}{n}\right)$ for t.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Holy Name, Hudson, Westborough

ROUND III: Logic problems

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. K is a 3-digit whole number. Its leftmost digit is the largest possible, considering that it is the difference of the two rightmost digits. Each digit is unique The middle digit is as large as possible and the rightmost digit is as small as possible. Find K.

2. Dave and Sue's mother is pregnant. Dave says "If it's a girl, I'll have twice as many sisters as brothers." His sister Sue says "If it's a boy, I'll have twice as many brothers as sisters." How many children does their mother presently have?

3. On each play of a game any one of 5, 4, 3, 2, or 0 points can be scored. Find the number of combinations of these scores which yield a total of 30 points in 7 plays.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Bromfield, Hudson, Tahanto

ROUND IV: Number theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If the sum of 3 different prime numbers is an even number, what is the smallest of the three?

2. Find the product of the least common multiple and the greatest common factor of 24, 30, and 45.

3. Find the value of digit A if the 5-digit number 12A3B is to be divisible by both 4 and 9, with $A \neq B$.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Bromfield, Hopedale, South

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET 3 points each

1. Using a=1, b=2, c=3, d=4, e=5, and f=6,
   a) evaluate this expression by following the customary order of operations.
      • means multiply.

      $$a + b \cdot c - d + e \cdot f$$

   b) evaluate the same expression but by doing each operation in order left to right.
   Both answers must be right for credit.

2. Solve: $-x - \left\lceil \frac{1}{2}(3x-1) + \frac{1}{4} \right\rceil = \frac{5x}{4} - 2$

3. A, B, C, and D represent 4 different digits. How many solutions of

$$\begin{array}{r} 3 \\ +A \\ \hline B \end{array} \qquad \begin{array}{r} B \\ +C \\ \hline D3 \end{array}$$

   are there?

4. In what bases, 3 through 12 inclusive, is 2101 a perfect square in base 10?

5. According to medical statistics, the average birth weight of a baby in the U. S. is 7.5 lbs. After birth most babies lose an average of one ounce a day for the first five days and then gain an avarage of one ounce a day for the next 13 weeks. If a baby weighs 7.5 lbs at birth, after how many days will the baby weigh 12 pounds? 1 pound = 16 ounces.

6. You have 6 coins in your pocket: 2 dimes, 3 quarters, and 1 nickel. What is the probability that you will pick a quarter both times if you pick one coin, return it, and pick again? Assume that each coin is equally likely to be picked each time.

7. All the even numbers from 2 through 98 inclusive, except those ending in 0, are multiplied together. What is the units digit of the product?

8. A hotly debated issue in the enchanted forest is whether to seed the clouds in order to produce more rain. There is a definite difference of opinion among the various types of trees. The f[illegible] s want any vote to be equitable and realize that some types of trees are more numerous than others. The leaders have set up this system: 6 pine tree votes are equal to 9 oak tree votes, 4 beech tree votes = 13 maple tree votes. and 6 beech tree votes = 8 pine tree votes. What would be the equation relating maple tree votes and oak tree votes?

November 3, 1999 WOCOMAL Freshman Meet ANSWERS

ROUND I eval

1. 1 pt 17
2. 2 pts -5.5
3. 3 pts $\frac{4}{27}$

ROUND II solve linear eq

1. 1 pt -2
2. 2 pts 5
3. 3 pts $\frac{n}{c}(c-v)$ or variations

ROUND III logic prob

1. 1 pt 891
2. 2 pts 6
3. 3 pts 6

ROUND IV # thry

1. 1 pt 2
2. 2 pts 1080
3. 3 pts 1

TEAM ROUND 3 pts each

1a) 33 b) 60 need both

2. $\frac{3}{5}$ OR .6
3. 4
4. 3 and 8 need both
5. 82 days
6. $\frac{1}{4}$ OR 25%
7. 6
8. 13 maples = 8 oaks

November 3, 1999 WOCOMAL Freshman Meet BRIEF SOLUTIONS

ROUND I

1. $2 \clubsuit 3 = 2^3 + 3^2 = 8+9 = 17$

2. $\left|\frac{3\cdot 2}{-4}\right| - \left(-\left|\frac{4}{2}\right|\right) - 9 = \frac{3}{2} + 2 - 9 = -5\frac{1}{2}$

3. $\frac{a^2+b^2+2ab}{a+b} = \frac{(a+b)^2}{a+b} = a+b$

get $\frac{1}{27} + \frac{1}{9} = \frac{4}{27}$

ROUND II

1. $2-4y-6+8y-10-12y+14 = 16$

$-8y = 16 \Rightarrow y = -2$

2. Mult by 40 to get

$5(7x+5) - 4(3x+15) = 80$

$35x+25-12x-60 = 80$

$23x = 115 \Rightarrow x = 5$

3 Mult by n to get

$Vn = c(n-t)$

$Vn = cn - ct$

$ct = cn - Vn$

$t = \frac{cn-Vn}{c}$ or several other forms

ROUND III

1. middle digit is 9
rightmost is 1
leftmost is $9-1=8$
} number is 891

ROUND III cont.

2. Experiment for a while and find that 3 boys, 3 girls now works. Hope that this is unique. Ans. 6

OR

If g, $2(b-1) = g+1$
If b, $2(g-1) = b+1$
} $\Rightarrow g = b = 3$
Ans. 6

3
5, 5, 5, 5, 5.5, 0
5.5, 5.5, 5, 3, 2
5, 5, 5.5, 4 4, 2
5, 5, 5.5, 4, 3, 3
5, 5, 5, 4, 4, 4, 3
5, 5, 4, 4, 4, 4, 4

Ans 6

ROUND IV

1 Can't have 3 odds (odd sum). Smallest prime, 2, thus is it

2 LCM of $2^3 \cdot 3$, $2 \cdot 3 \cdot 5$, and $3^2 \cdot 5$ is $2^3 \cdot 3^2 \cdot 5 = 360$

GCF is 3. Ans $3 \cdot 360 = 1080$

3. For 12A3B to be divisible by 4, 3B must be 32 or 36, so $B = 2$ or 6.

For divisibility by 9, $1+2+A+3+B$ must be a multiple of 9.

If $B = 2$, $A = 1$ and if $B = 6$, $A = 6$.

But $A \neq B$, so $A = 1$ only

TEAM ROUND

1a. $1+2\cdot3-4+5\cdot6 = 1+6-4+30 = 33$

b. Get $3\cdot3=9$, then 5, 10, 60

2. Mult by 4 to get

$$-4x-2(3x-1)-1 = 5x-8$$
$$-6x+2 = 9x-7$$
$$9 = 15x \Rightarrow x = \frac{3}{5}$$

3.
$$\frac{3}{1}\,\frac{4}{9}\,\frac{3}{2}\,\frac{5}{8} \qquad D=1$$
$$\frac{4}{4}\,\frac{13}{13}\,\frac{5}{5}\,\frac{13}{13}$$

$$\frac{3}{4}\,\frac{7}{6}\,\frac{3}{6}\,\frac{9}{4} \qquad \text{Ans } 4$$
$$\frac{7}{7}\,\frac{13}{13}\,\frac{9}{9}\,\frac{13}{13}$$

4. Try them all

$2101_3 = 64 = 8^2$

$2101_8 = 1089 = 33^2$

Ans 3 and 8

5. $7.5 - 5\cdot\frac{1}{16} + (n-5)\frac{1}{16} = 12$

$$-\frac{5}{16} + \frac{n}{16} - \frac{5}{16} = 4\tfrac{1}{2}$$
$$\frac{n}{16} = \frac{9}{2} + \frac{10}{16}$$
$$n = 72+10 = 82 \text{ days}$$

6. 6 coins available, 3 are quarters.

$P(\text{quarter each time}) = \frac{3}{6} = \frac{1}{2}$

$P(\text{quarter both times}) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

7. $2\cdot4\cdot6\,8$ ends in 4.

4^{10} ends in 6 (calculator) $\therefore$ 6

8. $6p = 9o \Rightarrow p = \frac{3}{2}o$

$4b = 13m \Rightarrow m = \frac{4}{13}b$

$6b = 8p \Rightarrow b = \frac{4}{3}p = \frac{4}{3}\left(\frac{3}{2}o\right) = 2o$

$m = \frac{4}{13}(2o) \Rightarrow 13m = 8o$